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Performance of Fourier-Based and Wavelet-Based OFDM for DVB-T Systems

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***Abstract*— We present a comparative study on Fourier-based OFDM (FFT-OFDM) and wavelet-based OFDM (DWT-OFDM) in DVB-T system (DWT being the discrete wavelet transform). We found that the DWT-OFDM outperforms FFT-OFDM in AWGN and Rayleigh fading channels. For AWGN channel, the gain in term of energy per bit to noise ratio Eb/No was improved by about 5 dB when the system used Haar wavelet compared to FFT-OFDM with a cyclic prefix (CP) of 1/4-th the total OFDM symbol period, for the same BER of 0.001. Other members of Daubechies families such as db8, db16 and db32 also outperformed by the gains of 7 dB, 10 dB and 11 dB, respectively, at the same BER. We also considered Daubechies wavelet db32 and FFT-OFDM with a CP of 1/4-th the total OFDM symbol period in the presence of narrowband interference. In terms of Eb/No, DWT-OFDM surpassed FFT-OFDM by 9 dB at 0.02 BER. It is also shown that the DWT-OFDM of Daubechies db8 and db1 outperform FFT-OFDM in Rayleigh fading, both in multipath flat fading and multipath frequency selective fading. The DWT-OFDM's with db8 and Haar outperformed FFT-OFDM by 7 and 2 dB, respectively, at BER of 0.01 in the flat fading channel. In addition, DWT-OFDM and FFT-OFDM showed about the same performance below 10 dB of Eb/No in frequency selective fading. However, the wavelet-based OFDM showed significant improvement of performance at higher than 10 dB of Eb/No.**

***Index Terms*— Wavelet, DWT, Fourier, OFDM, DVB-T.**

I. Introduction

An OFDM system is a multi-carrier system which processes signals to be transmitted in parallel at different frequencies simultaneously from the same source. Conventional OFDM system used IFFT and FFT to multiplex the signals in parallel with reduced complexity algorithm at the transmitter and receiver respectively. The system employs guard interval or cyclic prefix (CP) so that the delay spread of the channel becomes longer than the channel impulse response. The reason is to minimize inter-symbol interference between symbols. However, the CP brings the disadvantage to the spectral containment of the channels [1]. Many researchers have investigated the use of wavelet based to replace Fourier based OFDM and found out that the wavelet based has more advantages than Fourier based OFDM [1]-[4]. Some researchers have made comparisons between discrete multitone (DMT) and discrete wavelet multitone (DWMT) systems. However, they have not considered about the applications of digital video broadcasting-terrestrial (DVB-T) in their investigations. In our simulation, the use of DVB-T in which that the bandwidth of the transmission is 8 MHz compliance with ETSI standard has been considered. We made comparisons based on BER Monte Carlo simulation for AWGN and Rayleigh fading channels.

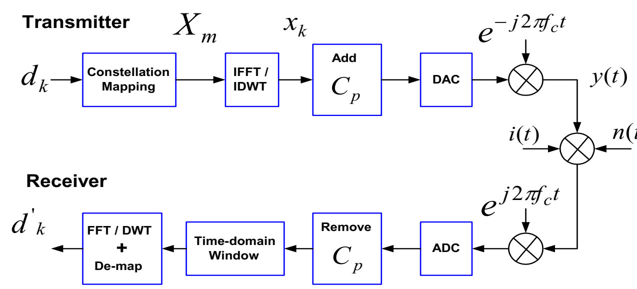

Fig. 1. An OFDM transceiver.

The organization of this paper is as followed: the descriptions of Fourier based and wavelet based OFDM are discussed in section II and III respectively, the system analysis is included in section IV and finally the system simulation of performance is discussed in Section V.

II. Fourier-Based OFDM

In Fig. 1, the data $\{d_k\}$ is processed by M-ary QAM modulator to map the data before IFFT, with N subcarriers. Its output is the sum of the information signals in the discrete time bearing as following:

$$x_k = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m e^{j2\pi km/N} \tag{1}$$

where $\{x_k | 0 \leq k \leq N-1\}$ is a sequence in the discrete time domain, $\{X_m | 0 \leq m \leq N-1\}$ are complex numbers in discrete frequency domain. The cyclic prefix (CP) is added before transmission to minimize the inter-symbol interference. At the receiver side, the processed is reversed to obtain and decoded the data. The CP is removed to obtain the data in discrete time domain. The data is then processed to the Time-Domain(TD) windowing for eliminating the narrowband interference before FFT. The output of FFT is the sum of the received signal in discrete frequency domain as follows:

$$X_m = \sum_{k=0}^{N-1} x_k e^{-j2\pi km/N} \tag{2}$$

III. Wavelet-Based OFDM Description

The wavelet transform blocks, inverse discrete wavelet transform (IDWT) and discrete wavelet transform (DWT) have replaced the IFFT and FFT in modulation and demodulation of FFT-OFDM system. Due to the overlapping nature of wavelet properties, the wavelet based does not need cyclic prefix to deal with delay spreads of the channel. As a result, it has higher spectral containment than that of Fourier-based OFDM [1].

The data $\{d_k\}$ is processed as per FFT-OFDM. However, the difference is that the system does not require CP to be added to the OFDM symbol, and the system uses inverse discrete wavelet transform (IDWT) and discrete wavelet transform (DWT) to replace IFFT and FFT in transmitter and receiver, respectively. The output of the inverse discrete wavelet transform (IDWT) can be represented as [5]:

$$s(k) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} S_m^n 2^{m/2} \psi(2_k^m - n) \quad (3)$$

where $\{S_m^n\}$ are the wavelet coefficients and $\psi(t)$ is the wavelet function with compressed factor m times and shifted n times for each subcarrier (number k, $0 \leq k \leq N-1$). The wavelet coefficients are the representation of signals in scale and position or time. The scale is related to the frequency. Low scale represents compressed wavelet which means that the signal is rapidly changing, or the signal is in high frequency. On the other hand, high scale represents stretched wavelet which means that the signal is slowly changing, or the signal is in low frequency. Thus, X_m can be represented to $\{S_m^n\}$ before it is processed to IDWT. At the receiver side, the process is inversed. The output of discrete wavelet transform (DWT) is

$$S_m^n = \sum_{k=0}^{N-1} s(k) 2^{m/2} \psi(2_k^m - n) \quad (4)$$

S_m^n can be decoded to X_m before the recovery of data to QAM demodulator.

An example of a baseband signal is shown in Fig. 2. In this figure, the top plot is the output signals from QAM modulator. Other plots are the Daubechies' wavelets OFDM samples in discrete time that are transmitted after an IDWT block. The plots are shown for the first 100 samples out of 1705 samples.

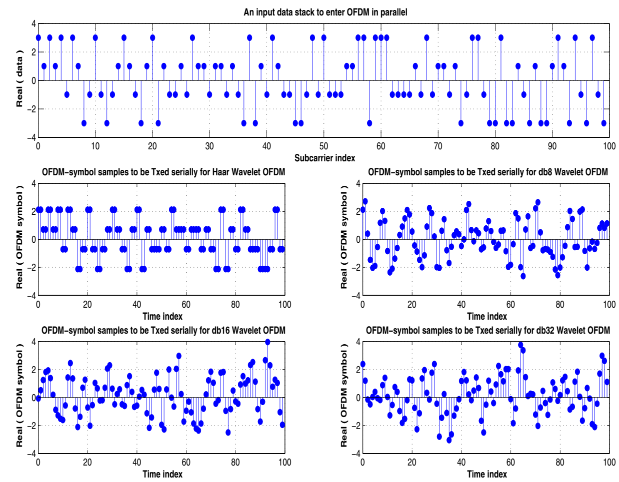

Fig. 2. Baseband signals of the Daubechies' wavelets showing the first 100 samples in an OFDM transmitter(number of subcarriers: 1705). Top: The Modulating Signals from QAM. Second Row(left): Haar or db1 Wavelet OFDM. Second Row(right): db8 Wavelet OFDM. Third Row(left): db16 Wavelet OFDM. Third Row(right): db32 Wavelet OFDM.

IV. SYSTEM ANALYSIS

A. Fourier-Based OFDM (FFT-OFDM)

The system block diagram in Fig. 1 assumes that there is no frequency offset at the receiver. Thus, the FFT itself acts as match filtering. To determine the data in sub-channel k, we match the transmitted waveform with carrier i [4]:

$$\langle y(t), f_i(t) \rangle = \sum_{k=0}^{K-1} d_k \langle f_k(t), f_i(t) \rangle \quad (5)$$

where $y(t)$ is the transmitted data via IFFT, $f_k(t)$ complex exponentials used in the IFFT operation or $e^{j2\pi km/K}$ (K being the size of FFT), d_k the data projected on each carrier, $\langle f_k(t), f_i(t) \rangle$ equals 1 when $k = i$ and 0 when $k \neq i$.

In a typical communication system, data is transmitted over a dispersive channel. The impulse response of a deterministic (and possibly time-varying) channel can be modelled by a linear filter $h(t)$:

$$\begin{aligned} r(t) &= y(t) * h(t) + n(t) \\ &= \sum_{k=0}^{K-1} d_k f'_k(t) + n(t) \end{aligned} \quad (6)$$

where $f'_k(t) = f_k(t) * h(t)$ or $f'_k(t)$ being the distorted carriers due to the dispersive channel and n(t) is additive white Gaussian noise. When matching the transmitted waveform with carrier i, we have

$$\begin{aligned} \langle y(t), f_i(t) \rangle &= \sum_{k=0}^{K-1} d_k \langle f'_k(t), f_i(t) \rangle + \langle n(t), f_i(t) \rangle \\ &= \sum_{k=0}^{K-1} d_k \rho_{k,0}(0) + \langle n(t), f_i(t) \rangle \\ &= d_K \rho_{i,i}(0) + \sum_{\substack{k=0 \\ k \neq i}}^{K-1} d_k \rho_{k,i}(0) + n''(t) \end{aligned} \quad (7)$$

where $d_K \rho_{i,i}(0)$ is the recovered data with correlation term $\rho_{i,i}(0)$ and $n''(t)$ is uncorrelated Gaussian noise. The interference term $i(t) = \sum_{k=0,k\neq i}^{K-1} d_k \rho_{k,i}(0)$ degrades the system performance. It causes the filter to be distorted and it is no longer orthogonal to one another with correlation terms $\rho_{k,i}(0)$. If the channel has no distortion, this term becomes 0 and would yield to possibly decode exactly what was transmitted plus a Gaussian noise term.

B. Wavelet-Based OFDM (DWT-OFDM)

In wavelet-based OFDM, same analysis using match filtering is performed except that the $f_k(t)$ and $f_i(t)$ are replaced with $W_k(t)$ and $W_i(t)$; $W_k(t)$ being the wavelet carrier in

IDWT operation with k sub-channels to match with carrier i. Thus, the received signal is as follows:

$$\begin{aligned} r_W(t) &= y_W(t) * h(t) + n(t) \\ &= \sum_{k=0}^{K-1} d_k W'_k(t) \\ &+ \sum_{l=0}^{g-1} \sum_{k=0}^{K-1} d_{k,l} W'_k(t-lk) + n(t) \end{aligned} \quad (8)$$

where K is the wavelet filter rank (sampling rate), $W'_k(t) = W_k(t) * h(t)$, and g ($g > 1$) is the wavelet *genus* so that Kg is the filter order (number of taps in that sub-band). After matched - filtering with carrier i, the signal becomes

$$\begin{aligned} \langle r_W(t), W_i(t)\rangle &= \sum_{k=0}^{K-1} d_k \langle W'_k(t), W_i(t)\rangle \\ &+ \sum_{l=1}^{g} \sum_{k=0}^{K-1} d_{k,l} \langle W'_k(t-lk), W_i(t-lk)\rangle \\ &+ \langle n(t), f_i(t)\rangle \\ &= \sum_{k=0}^{K-1} d_k \rho_{k,0}(0) + \langle n(t), f_i(t)\rangle \\ &= d_K \rho_{i,i}(0) + \sum_{\substack{k=0\\k\neq i}}^{K-1} d_k \rho_{k,i}(0) \\ &+ \sum_{l=1}^{g} \sum_{\substack{k=0\\k\neq i}}^{K-1} d_{k,l} \rho_{k,i}(l) + n''(t) \end{aligned} \quad (9)$$

where $d_K \rho_{i,i}(0)$ is the recovered data with correlation term $\rho_{i,i}(0)$. The term $\sum_{k=0,k\neq i}^{K-1} d_k \rho_{k,i}(0)$ is the interference due to the distorted filters that are no longer orthogonal to one another with correlation terms $\rho_{k,i}(0)$, and $\sum_{l=1}^{g} \sum_{k=0,k\neq i}^{K-1} d_{k,l} \rho_{k,i}(l)$ is the interference term with correlation $\rho_{k,i}(l)$ due to the overlapped nature of wavelet transform. These two terms become 0, and only the first and last terms would appear if the channel has no distortion. The decoder would possibly obtain almost the correct signal when the two terms are zero.

V. SYSTEM SIMULATION

In this section, the performance of wavelet-based OFDM is compared with Fourier-based OFDM. Assumptions are made that the simulation is performed without the consideration of the channel equalization or channel estimation. For the system simulation, the OFDM parameters for DVB-T system are used with compliance of ETSI standard. This is shown in Table I. Note that the term 2k mode refers to the number 1705 of subcarriers in ETSI standard above.

Fig. 3 shows the frequency response of the system at the front end receiver of OFDM (DVT-B system) in AWGN channel. The top part shows the signal with carrier frequency about 90 MHz. The bottom part shows the signal if there is an unwanted signal or an interference within the same 8 MHz bandwidth. In this case, we showed the unwanted signal having the carrier frequency of about 91 MHz. To simulate the signal to satisfy the transmission bandwidth of 8 MHz for DVT-B system, the sampling frequency has to satisfy Nyquist criterion, considering at least twice of the carrier frequency. In this simulation, we used a carrier frequency of about 90 MHZ for VHF channel, making the sampling frequency (f_s) of at least 180 MHz or the sampling time (t_s) was reciprocal of it, about 5.47 nano seconds.

The Daubechies' DWT-OFDM family outperform the Fourier-based FFT-OFDM as shown in Fig. 4. In this simulation, we used CP of 25% of the total OFDM symbol period for the FFT-OFDM system. The DWT-OFDM families do not require cyclic prefix due to the overlapping nature of their properties. The Haar or db1 wavelet outperformed the FFT-OFDM by E_b/N_o margin of 5 dB, for the same BER of

TABLE I
OFDM PARAMETERS FOR THE 2K MODE FROM [6].

Parameter	2k mode
Duration OFDM symbol period, T_u	224e-6
Baseband elementary period, T_b	T_u/2048
Number of carriers K	1705
Value of carrier number $K_{\max}$	1704
Value of carrier number $K_{\min}$	0
Carrier Spacing $1/T_u$	4464 Hz
Spacing between carriers $K_{\max}$ and $K_{\min}$, $(K-1)/2$	7.61 MHz
Allowed guard interval	1/4 1/8 1/16 1/32
Duration of symbol part T_u	$1048 \times T_b = $ 224e-6
Duration of guard interval Δ	$512 \times T_b = $ 56e-6 $256 \times T_b = $ 28e-6 $128 \times T_b = $ 14e-6 $64 \times T_b = $ 7e-6
Symbol duration $T_s = \Delta + T_u$	$2560 \times Tb = $ 280e-6 $2304 \times T_b = $ 252e-6 $2176 \times T_b = $ 238e-6 $2112 \times Tb = $ 231e-6

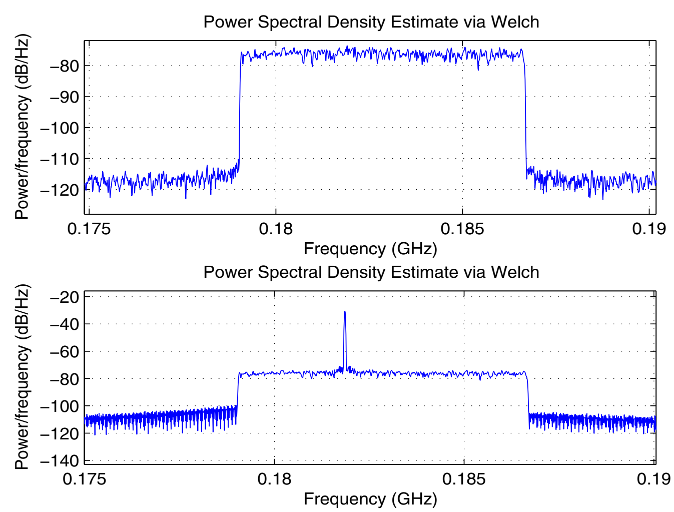

Fig. 3. Frequency response of 2k mode signal (number of subcarriers: 1705) at the front-end receiver of (FFT-16-QAM with CP 1/4 th of symbol period) OFDM (DVT-B system) in AWGN channel without (Top figure) and with (Bottom) interference.

0.001 on AWGN channel. Other members of the Daubechies family such as db8, db16, and db32 also outperformed the FFT-OFDM by 7 dB, 10 dB, and 11 dB, respectively, at the same BER.

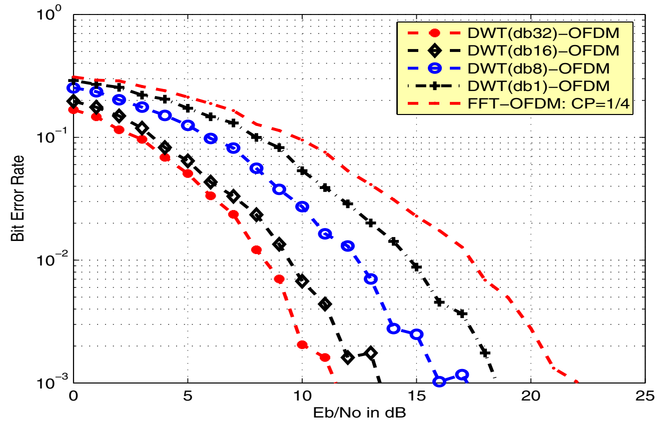

Fig. 4. Performance of Bit error rate (BER) of Fourier-based OFDM and different Daubechies DWT-OFDM's over AWGN channel using 16-QAM.

In the presence of narrowband interference, the performance of OFDM in terms of E_b/N_o was also shown. In Fig. 5, the comparison was made when the narrowband interference may co-exist within the DVB-T spectrum and also when it was absence for both systems, the wavelet-OFDM (db32-OFDM) and FFT-OFDM with 25% of the total OFDM symbol period. Assumption was made that the carrier frequency of the narrowband interference was 91 MHz next to the OFDM transmission frequency carrier for both systems. We used a simple method, time windowing technique as mentioned in [7] to suppress the interference. The wavelet-based OFDM with db32 showed a significant performance improvement of about 6 dB at BER of 0.01 over FFT-OFDM when the suppression method was applied.

A comparison of BER was also observed in a multipath flat-fading channel in Fig. 6. We considered that the channel is static or the maximum Doppler shift is 0. The DWT-OFDM's with db8 and Haar outperformed Fourier-based OFDM by 7 dB and 2 dB, respectively, at BER of 0.01. Performance in multipath frequency-selective fading was also simulated. In this case, we assumed that the receiver is a pedestrian with a walking speed of 1 m/s in an urban area. Using the formula $f_d = (v * f_c)/C$ ($C = 3 \times 10^8$ m/s), a maximum doppler shift (f_d) of about 0.3 Hz was obtained. The difference in time between path delays was approximately 6 micro seconds. In Fig. 7, the performance curves of Haar wavelet and db8 were almost the same as that of FFT-OFDM at low E_b/N_o (less than 10 dB). However, the results showed significant improvement by DWT-OFDM with E_b/N_o higher than 10 dB. The system might be further improved at low E_b/N_o if we could use a single-tap channel equalizer to compensate the performance due to the Rayleigh fading.

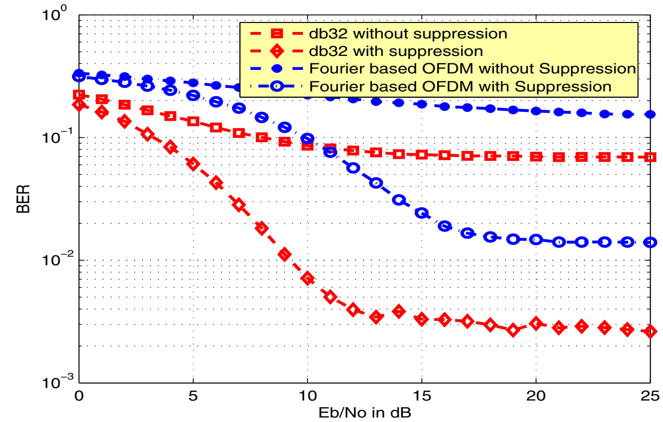

Fig. 5. Performance of BER of FFT-OFDM and db8/db1 (Haar) - OFDM over AWGN channel using 16-QAM in the presence of narrowband interference.

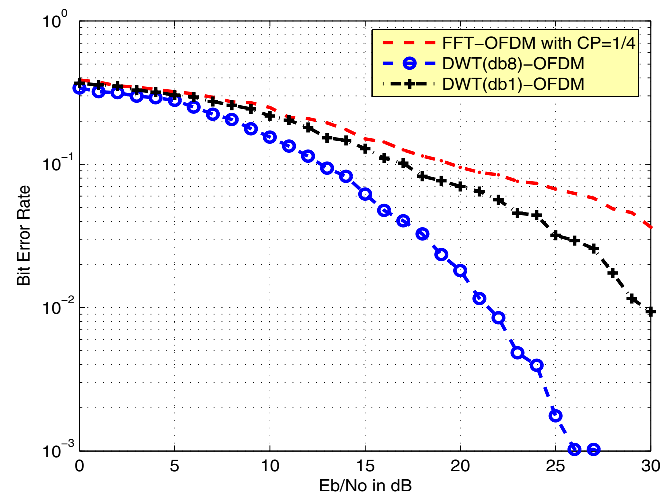

Fig. 6. Performance of BER of FFT - OFDM and db8/db1 Haar -OFDM and over multipath fading.

VI. Conclusion

It is shown that the wavelet-based OFDM (DWT-OFDM) out-performs Fourier-based OFDM (FFT-OFDM) in terms of E_b/N_o for the same bit error rate (BER) target of 0.001 in DVB-T system. As the order of Daubechies filter increases from 1 (haar) to 32, the E_b/N_o gain also increases and all showed improvement over Fourier-based OFDM. In the presence of narrowband interference, the performance of OFDM in terms of E_b/N_o was also shown. The wavelet-based OFDM with db32 showed a significant performance of improvement of about 6 dB at BER of 0.01 over FFT-OFDM when the time windowing technique was applied. Further simulation were also done in Rayleigh fading channels under multipath flat-fading and frequency - selective fading. In both cases, the performance of the DWT-OFDM also shows improvement over the performance of FFT-OFDM in DVB-T system.

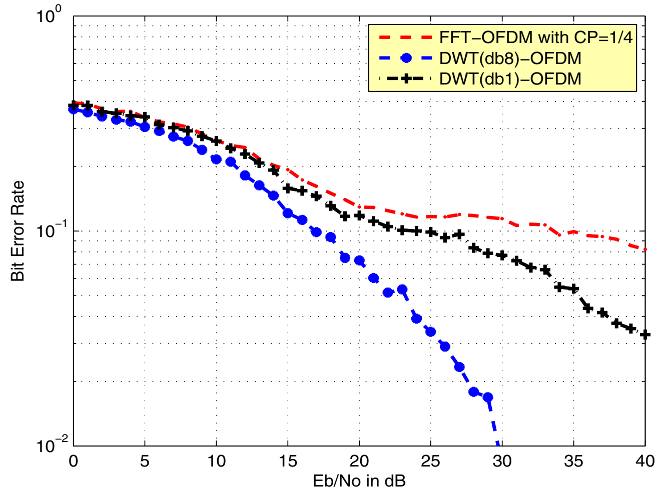

Fig. 7. Performance of BER of FFT-OFDM and db8/Haar-OFDM over multipath (i.e multipath follows Rayleigh fading)

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